

Application Note (A9)

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## Improving Integrating Sphere Design for Near-perfect Cosine Response

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# IMPROVING INTEGRATING SPHERE DESIGN FOR NEAR-PERFECT COSINE RESPONSE

## Introduction.

Integrating devices generally fall into two categories: transmissive diffusers and reflective diffusers. Transmissive devices, commonly referred to as "cosine diffusers", rely on scattering materials and shape to give their cosine response. However, scattering is strongly dependent on the wavelength of light so the bluer wavelengths are scattered more strongly than redder. This leads to the generally observed strong wavelength dependence of cosine response for such devices. Transmissive diffusers can be optimized for near-perfect cosine response, but only at one wavelength, and suffer from dramatically reduced throughput at shorter wavelengths and increasingly poor response at longer. Integrating spheres show little wavelength effect but generally have poorer cosine response. It is the purpose of this article to show that poor cosine response is not an inherent property of integrating spheres, but rather the poor designs currently used. With good design, integrating spheres can be virtually ideal devices: near-perfect cosine response with negligible wavelength effects.

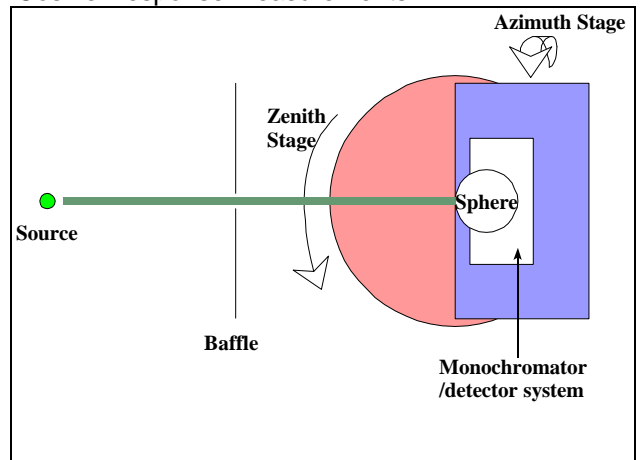
Many studies in radiometry and photometry require measurements of light falling onto a flat surface. Such studies vary from solar UV exposure limits to comfortable office lighting conditions, yet all are dependent on the cosine response of the measurement device. A poor cosine response leads to large errors whenever the size, shape or orientation of the test source is different to the calibration source. Since these are systematic errors, and are extremely difficult to quantify, it is best to eliminate them as far as practicable.

Several types of cosine error exists: zenith angle, azimuth angle and spectral. These are generally considered independent, so errors should be determined at each zenith angle, azimuth angle and wavelength to totally describe the device. With so many measurements required to describe the cosine response of a device it is not surprising that manufacturers often supply only very basic information.

## Measurement of Cosine Response.

The correct measurement procedure is important in determining the cosine response accurately. The setup, shown in figure 1, consists of a distant source or uniform collimated beam, an appropriate baffle, zenith and azimuth angle rotary stages with the center of the sphere's input aperture at the center of rotation, and a monochromator/detector system to analyze spectral components. Zero degrees zenith angle is defined by reflection of the incident beam by a mirror, parallel to the entrance aperture, back to the source. Zero

**Figure 1** - Experimental arrangement for accurate Cosine Response Measurements



degrees azimuth is arbitrary, but here we adopt the most common definition for spheres of right-angle geometry (i.e. the input and exit ports are at 90 degrees apart) as "the plane containing the centers of the sphere, entrance and exit apertures".

### **Modeling Sphere Responses.**

Measurement of the various combinations of zenith angle, azimuth angle and wavelength is an arduous task requiring many hours or days of effort. If the sphere response could be accurately modeled, and verified experimentally only at appropriate points, then a detailed map of the cosine response could be generated with minimal resources. Moreover, if the cosine response is calculated from physical characteristics of the sphere, then modification of those characteristics to improve response will be an invaluable design aid.

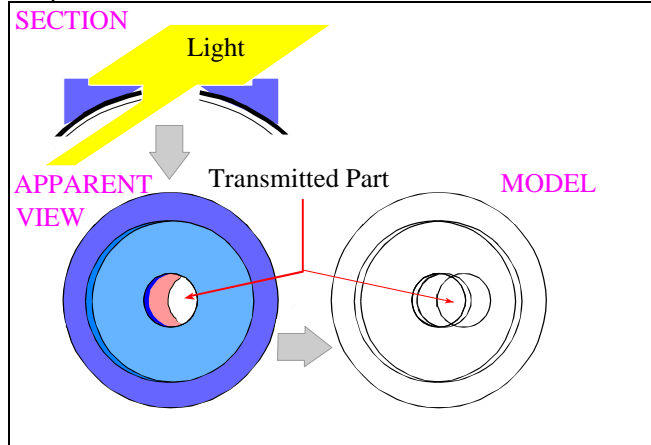
The first stage of this model concerns the input aperture design. It is a truism to say that if light entering the sphere is not cosine dependent then little can be done inside the sphere to compensate. However, this is precisely what happens in most current designs. Most spheres are made as just that i.e. spherical, and then coated on the inside with several millimeters of reflective material. This means that two apertures now exist: one in the original sphere and one at the inside of the coating. Also, the thickness of the sphere walls and any fixtures around the input aperture can contribute additional apertures. When

considering the sphere response due to input aperturing, they can be treated as a series of circles whose centers move relative to one another as shown in figure 2. By calculating the clear aperture area relative to that at zero zenith angle, the attenuation due to input aperturing at each zenith angle can be evaluated.

Once the light is inside the sphere it should be integrated to remove all directional effects. However, since light can also escape from the input aperture and the angle subtended from where the beam hits the sphere to the input aperture also varies with angle, we may expect a contribution from these losses to affect the overall cosine response of the sphere. To model this effect as simply as possible, several justifiable assumptions are made.

- i. The clear input aperture is small compared to the size of the sphere.
- ii. The angle and distance to the input aperture is the same over the whole of the illuminated area. This should hold true providing assumption i. is correct, except when the illuminated area is close to the input aperture, but at these large zenith angles aperturing effects tend to dominate anyway.

**Figure 2** - A sphere section and apparent view showing the effect of input aperturing on sphere response.



iii. The sphere coating has a constant, Lambertian, reflectance.

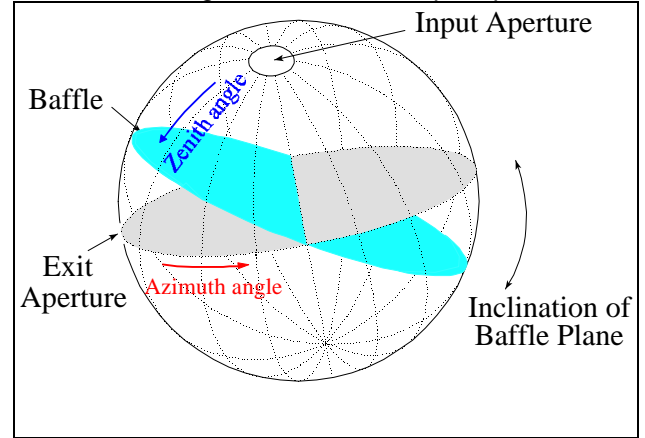
To implement this model, two separate calculations are made: one for the losses from a sphere, and the other for losses from a plane surface passing through the center of the sphere (representing the baffle) as shown in figure 3. Using appropriate combinations of plane and spherical trigonometry at each zenith and azimuth angle, the solid angle subtended to the input aperture, as a fraction of  $2\pi$  steradians, is multiplied by the cosine of the angle from the center of the illuminated area to the center of the input aperture. This value is therefore the fractional loss from the sphere at those angles. Since values are calculated as fractions of the total, any variations in the illuminated area cancel out.

By projecting the illuminated area of the plane onto the actual dimensions of the baffle, as shown in figure 4, the fractions of light hitting the baffle and sphere can be determined. First, the central positions, relative to the baffle, of the beam intersecting with the plane is determined. Next, the overlapping area of the beam and baffle, taking account of the change in the beam shape with angle, is determined as a fraction of the total beam area. Multiplying the losses from the plane and sphere by their respective fractions then gives the total loss at that zenith and azimuth angle. By subtracting this value from one, the fraction of light retained within the sphere, i.e. the responsivity taking this effect into account, is determined.

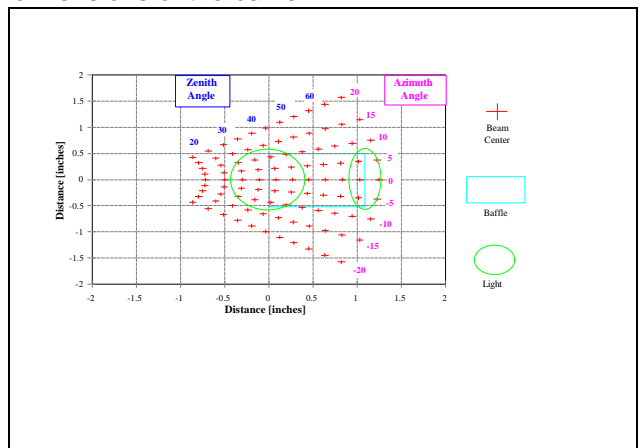
### Designing the Sphere.

Multiplying the responsivities of aperturing and losses from the input aperture by the cosine of the zenith angle gives the overall cosine response of the sphere. Modeling different input configurations, it is found that acceptable aperturing responsivity is only obtained by having the input port level with

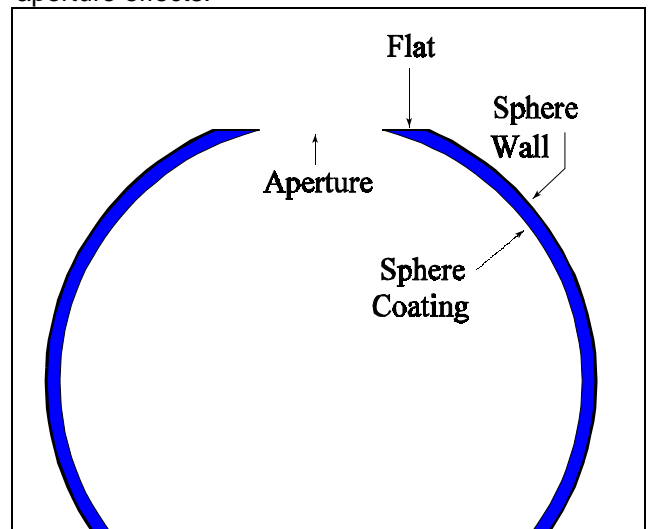
**Figure 3** - Diagram of the sphere and baffle plane used in modeling losses from the input aperture.



**Figure 4** - Mapping the plane for the actual dimensions of the baffle



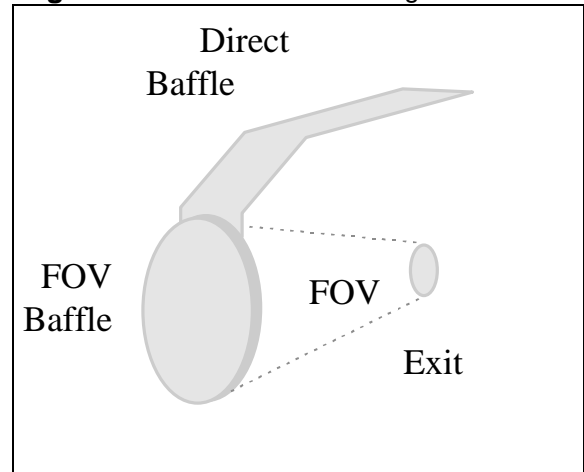
**Figure 5** - Input aperture design giving minimum aperture effects.



the inside of the sphere. This is achievable in practice by introducing a flat side to the sphere, as shown in figure 5.

The design of the baffle is also important for proper operation of the sphere. The exit port should not "see" any light directly from the input or from direct illumination of the sphere (the so-called "first strike") since both of these are dependent on the zenith and azimuth angle. The baffle should therefore cover all input/exit angles and be slightly larger than the field-of-view of any exit optics. A suitable baffle design satisfying these conditions is shown in figure 6. This baffle should not be placed too near the exit port, nor should it intercept the zero zenith angle beam of the sphere, so the actual size is likely to be proportional to the sphere diameter.

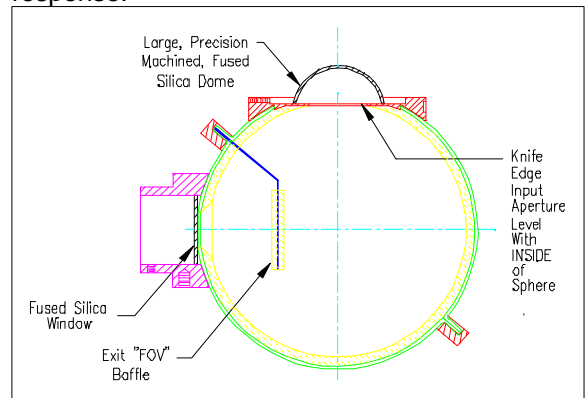
**Figure 6** - A suitable baffle design.



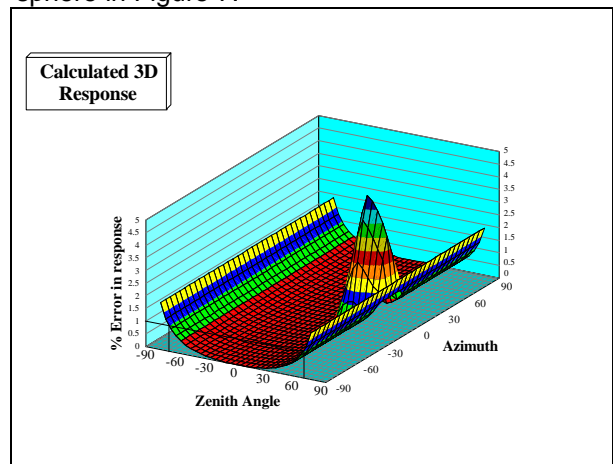
**Comparing Theory and Practice.**

These rather simple design ideas were used in modeling, and manufacturing, spheres of 4 inch and 6 inch diameter. Measurements of the cosine response on both spheres agreed well with predictions, and the results are presented for the 6 inch sphere. The sphere design, including mounting flanges for a dome window and exit attachments, is shown in figure 7. Using a spreadsheet, the responsivity at 5 degree intervals in both zenith and azimuth angles was calculated for this sphere. Since the results are close to ideal, they are best expressed as %error relative to ideal cosine response. Figure 8 shows a 3D plot of calculated %error for the different zenith and azimuth angles. It is readily seen that if this sphere were used for sunlight measurements, with the baffle facing South (in the Northern hemisphere), negligible cosine errors would be anticipated for all results where the zenith angle was less than 75 degrees.

**Figure 7** - A typical Young & Schneider integrating sphere design giving near ideal cosine response.



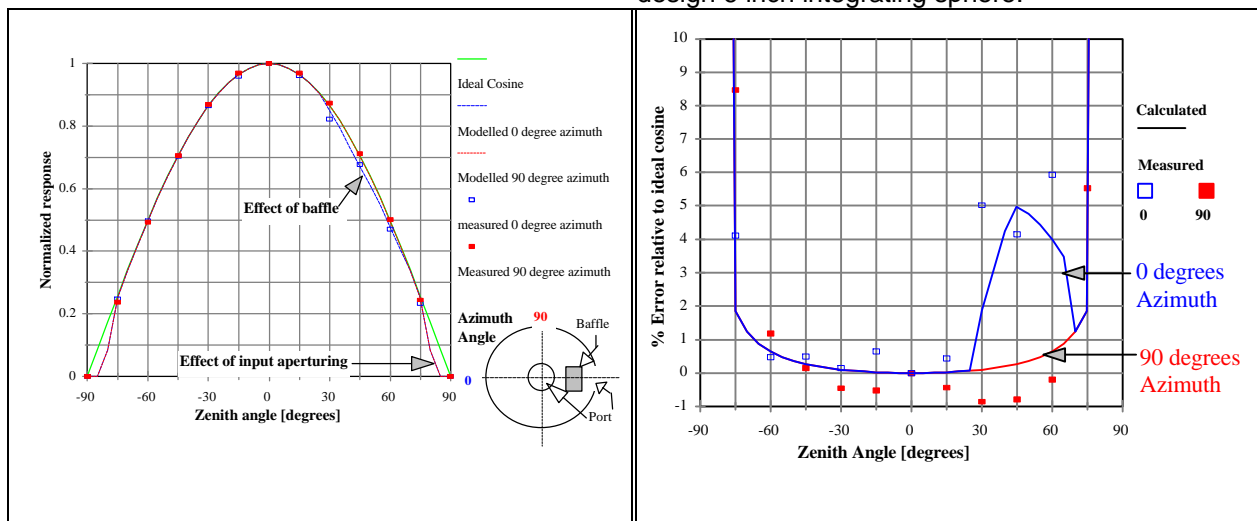
**Figure 8** - Calculated cosine errors for the sphere in Figure 7.



Measurements on this sphere, in 15 degree zenith angle steps, at 0 degree and 90 degree azimuth confirm that the calculated

values of the response (figures 9 and 10) are accurate. The measured response was determined at wavelengths of 300nm, 400nm, 500nm, 600nm, 700nm and 800nm for all angles. No differences beyond normal experimental error were seen, confirming the expected lack of wavelength dependence in the responsivity of the sphere. Some points near the baffle are some 2-3% different for measured and calculated responses. This is likely to be the result of multiple localized reflections, each of which loses light through the entrance port, before total randomization within the sphere is achieved. These localized multiple reflection effects, although calculable, are beyond the scope of this simple model and offer little significant improvement in accuracy.

**Figure 9** - Calculated and measured response of a Young & Schneider design 6 inch integrating sphere ideal cosine response for a Young & Schneider design 6 inch integrating sphere.



**Conclusion.**

Integrating spheres can be near perfect devices when designed properly, giving highly accurate cosine response at all wavelengths. The sphere response can be accurately estimated from relative simple formulae on a spreadsheet program, as verified by actual measurements. Moreover, full 3D characterization is possible by calculation, enabling components to be optimized and providing detail to the generally incomplete data provided by some manufacturers. Practical designs can benefit from this modeling, providing refinements in critical areas. A prime example, the Young & Schneider design, makes use of all of the optimization features and is commercially available from Optronic Laboratories, Inc.

**Related Studies.**

As mentioned earlier, global sunlight measurements require integrating devices with near-ideal cosine response. For further information, the reader should refer to the paper "The influence of cosine response on global sunlight measurements" by Young, Schneider and Austin.